FLA (Fall 2024) – Assignment 2

Name: ________________ Dept: ________________

Grade: ________________ ID: ________________

Due: Oct. 27, 2024

Problem 1

Prove that the following languages are not regular. You may use the pumping lemma and the closure properties of the class of regular languages.

a. $\{\omega 1 \omega \mid \omega \in \{0,\ 1\}^* \}$

b. $\{0^n 1^{2n} 2^{3n} \mid\ n \geq 0 \}$

c. $\{\omega \mid |0|_\omega \geq |1|_\omega,\ \omega \in \{0,\ 1\}^* \}$

d. $\{0^a 1^b \mid \gcd(a,\ b) = 2 \wedge a,\ b \geq 0 \}$ (**Hint**: Consider using factorial during string construction)

Proof.

Problem 2

Consider L_1 and L_2 as languages that are formed over the same alphabet Σ. The weave together of L_1 and L_2 is defined to be $W(L_1, L_2) = \{a_1b_1a_2b_2 \cdots a_nb_n | a_i, b_i \in \Sigma, a_1a_2 \cdots a_n \in L_1, b_1b_2 \cdots b_n \in L_2\}$. Prove that if L_1 and L_2 are regular, then $W(L_1, L_2)$ is also regular.

Proof.

Problem 3

Prove or disprove the following statements (All languages mentioned below are over alphabet Σ):

a. If A and B are not regular languages, then $A \cup B$ is not regular.

b. If A is not a regular language and B is a language such that $B \subset A$, then B is not regular.

c. If A is a language over alphabet Σ, h is a homomorphism on Σ and A is not regular, then $h(A)$ is not regular.

d. If A and B are not regular languages and C is a language such that $A \subseteq C \subseteq B$, then C is not regular.

Solution.

Problem 4

Let A and B be languages over $\Sigma = \{0, 1\}$. Define $N_0(w)$ is the number of 0s that string w contains and $N_1(w)$ is the number of 1s that string w contains.
Define:

$$A \sim_0 B = \{a \in A \mid \text{for some } b \in B,\, N_0(a) = N_0(b)\}$$
$$A \sim_{01} B = \{a \in A \mid \text{for some } b \in B,\, N_0(a) = N_0(b) \text{ and } N_1(a) = N_1(b)\}$$

a. Show that the class of regular languages is closed under $\sim_0$ operation.

b. Show that the class of regular languages is not closed under $\sim_{01}$ operation.

Solution.