

## FLA (Fall 2024) – Assignment 3

Name: \_\_\_\_\_ Dept: \_\_\_\_\_

Grade: \_\_\_\_\_ ID: \_\_\_\_\_

**Due: Nov. 10, 2024**

### Problem 1

Give context free grammars that generate the following languages, and give a brief description of the functionality of each variable in your grammars (in natural language) if necessary.

- a.  $\{w \in \{a, b\}^* \mid |a|_w > |b|_w, \text{ where } |x|_w \text{ denotes the number of occurrences of } x \text{ in string } w.\}$
- b.  $\{w \in \{a, b\}^* \mid abb \text{ and } aab \text{ are substrings of } w.\}$
- c.  $\{a^i b^j c^k \mid i, j, k \geq 0 \wedge (i = j \mid i = k).\}$
- d.  $\{a^i b^j c^k \mid i, j, k \geq 0 \wedge i = |j - k|.\}$

**Solution.**

## Problem 2

Let the language  $L$  consist of all the regular expressions over the alphabet  $\{0, 1\}$ . Thus,  $L$  is a language over the alphabet  $\{0, 1, \epsilon, \emptyset, +, \circ, *, (, )\}$  and includes, for example, the string  $(1 \circ (0 + 0 \circ 0))^*$  (For simplicity, let us assume that concatenation is always written out explicitly using the symbol  $\circ$ ).

Please construct two context-free grammars for  $L$ , one ambiguous and one unambiguous. For the ambiguous one, please give an example of a string with two different parse trees, and the corresponding left derivations.

**Solution.**

### Problem 3

Consider the following context free grammar  $G$ :

$$S \rightarrow 0AS \mid 0$$

$$A \rightarrow S1A \mid SS \mid 10$$

- a. For the string 0000100, give one possible parse tree and rightmost derivation according to  $G$ .
- b. Provide a nondeterministic PDA  $P$  that accepts the language  $L(G)$  by empty stack.

**Solution.**

## Problem 4

Begin with the grammar:

$$S \rightarrow BSA \mid AC \mid A$$

$$A \rightarrow aA \mid \epsilon$$

$$B \rightarrow Bba \mid b$$

$$C \rightarrow CC$$

- a. Eliminate  $\epsilon$ -productions.
- b. Eliminate any unit productions in the resulting grammar of (a).
- c. Eliminate any useless symbols in the resulting grammar of (b).
- d. Put the resulting grammar of (c) into Chomsky normal form.

**Solution.**