

FLA (Fall 2024) – Assignment 5

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: Dec.10, 2024

Problem 1

Consider the (deterministic) Turing machine M given by

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined in it, as described below.

1. $\delta(q_0, a) = (q_0, B, R)$
2. $\delta(q_0, b) = (q_1, B, R)$
3. $\delta(q_1, b) = (q_1, B, R)$
4. $\delta(q_1, B) = (q_2, B, R)$

Please answer the following questions:

- a. Specify the execution trace of M on the input string aab .
- b. Provide a regular expression for the language of the Turing machine.
- c. Suppose we added the transition $\delta(q_1, a) = (q_0, B, R)$ to the above machine, provide a regular expression for the language of the resulting Turing machine.

Solution.

Problem 2

Design single-taped single-tracked deterministic turing machines as follows:

- a. M_1 that computes function $f(x) = \begin{cases} x/2 & \text{if } x \text{ is an even} \\ (x+1)/2 & \text{if } x \text{ is an odd} \end{cases}$, where the input x is a positive binary integer (the high bits contain no extra zeros). When M_1 finishes running, only $f(x)$ is left on the tape.
- b. M_2 that decides the language $L = \{0^a 1^b 2^c \mid a = b = c\}$.

Solution.

Problem 3

Prove or disprove the following languages are decidable(not use Rice's Theorem):

- a. The set L of codes for TM's that never make a move left on any input.
- b. The set L of codes for TM's that, when started with the blank tape will eventually write some non-blank symbol on its tape. (Hint: If M has m states, consider the first m transitions that it makes)

Solution.

Problem 4

If a pushdown automaton has k stacks, we call it k -PDA. Clearly, 0-PDA is NFA, 1-PDA is PDA, and 1-PDA is more powerful than 0-PDA.

- a. What is the difference between the express ability of 2-PDA and 1-PDA? Please clarify your argument and prove the (un)equivalence.
- b. How about 2-PDA and regular TM? Give your explanation.
- c. How about 3-PDA and regular TM? Give your explanation.

Solution.