

FLA (Fall 2024) – Assignment 6

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: 24 Dec 2024

Problem 1

Answer the following statements **True** or **False**. If your answer is **True**, give an explanation. If your answer is **False**, give a counterexample. Please avoid using Rice's Theorem.

- a. \mathcal{P} is closed under **Kleene Closure**. So is \mathcal{NP} .
- b. \mathcal{P} is closed under **Complement**.
- c. $L = \{ \langle D \rangle \mid D \text{ is a DFA and } L(D) \text{ is finite} \}$ is decidable.
- d. $L = \{ \langle M \rangle \mid M \text{ is a Turing machine and } L(M) \text{ is finite} \}$ is decidable.

Solution.

Problem 2

We label the regular languages as A , the context-free languages as B , the recursive languages as C , the recursively enumerable languages as D , and the all possible languages as E . Please answer the most appropriate label of following languages. For example, the language of balanced parentheses can be labeled as E , but the correct answer is B . (We denote the complement of language L as \bar{L} .)

- a. The difference between a context free language and a regular language.
- b. The difference between a language in P and a recursive language.
- c. The complement of a language in NP.
- d. The complement of an undecidable language.
- e. $\{a^i b^j \mid i + 2j = 20\}$.
- f. $\{a^i b^j c^k d^l \mid i = j \text{ and } j = k\}$.
- g. $\{a^i b^j c^k d^l \mid i = l \text{ and } j = k\}$.

Solution.

Problem 3

Two space aliens walk into your home, both claiming to be oracles for the Boolean Satisfiability (SAT) decision problem. They both always give a yes/no answer in constant time for any SAT instance, and are each self-consistent (i.e. each always gives the same answer for the same instance). However, one is a true oracle and the other is a shameless impostor, and you have a large SAT instance F upon which they disagree (i.e. one claims that this SAT instance F is satisfiable and the other one claims that F is not satisfiable). Show that it is possible to expose the impostor within time polynomial in the size of that SAT instance F .

Solution.

Problem 4

A **useless state** in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a state in a Turing machine is useless. Formulate this problem as a language and show it is undecidable. (**Hint:** First, formulate the problem. Second, reduce to solve.)

Solution.